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Arc Flow Formulations Based on Dynamic Programming: Theoretical Foundations and Applications

Network flow formulations are among the most successful tools to solve optimization problems. Such formulations correspond to determining an optimal flow in a network. One particular class of network flow formulations is the arc flow, where variables represent flows on individual arcs of the network. For NP-hard problems, polynomial-sized arc flow models typically provide weak linear relaxations and may have too much symmetry to be efficient in practice. Instead, arc flow models with a pseudo-polynomial size usually provide strong relaxations and are efficient in practice.

The interest in pseudo-polynomial arc flow formulations has grown considerably in the last twenty years, in which they have been used to solve many open instances of hard problems. A remarkable advantage of pseudo-polynomial arc flow models is the possibility to solve practical-sized instances directly by a Mixed Integer Linear Programming solver, avoiding the implementation of complex methods based on column generation.

We present theoretical foundations of pseudo-polynomial arc flow formulations, by showing a relation between their network and Dynamic Programming (DP). This relation allows a better understanding of the strength of these formulations, through a link with models obtained by Dantzig-Wolfe decomposition.

The relation with DP also allows a new perspective to relate state-space relaxation methods for DP with arc flow models. We also present a dual point of view to contrast the linear relaxation of arc flow models with that of models based on paths and cycles.

To conclude, we review the main solution methods and applications of arc flow models based on DP in several domains such as cutting, packing, scheduling, and routing.